Gr 12 June P1 /150 2015

1.1. 1. $-x^2 - 5x = -6$

$0 = x^2 + 5x - 6$ ✔

✔ $= (x-1)(x+6)$

$\therefore x = 1$ or -6 ✔

3

2. $-3x^2 + 4x + 5 = 0$

$0 = 3x^2 - 4x - 5$

$= (\quad x \quad)$?

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✔

✔ $= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$

$= \dfrac{4 \pm \sqrt{76}}{6}$

✔ ✔ $= 2{,}12$ or $-0{,}79$

4

3. $\dfrac{2x^2}{x+2} \leq 2$

$\dfrac{2x^2}{x+2} - 2 \leq 0$

$\dfrac{2x^2 - 2(x+2)}{x+2} \leq 0$

$\dfrac{2x^2 - 2x - 4}{x+2} \leq 0$

✔ ÷2: $\dfrac{x^2 - x - 2}{x+2} \leq 0$ ✔

✔ $\dfrac{(x+1)(x-2)}{x+2} \leq 0$

$\ominus$ UD $+$ 0 $\ominus$ 0 $+$

$-2 \quad -1 \quad 2$

$x < -2$ or $-1 \leq x \leq 2$

✔ ✔

5

1.1. 4. $2^x - \dfrac{12}{2^x} = 4$

LCD $= 2^x \quad (\because 2^x \neq 0)$

x thru

✔ $2^{2x} - 12 = 4 \cdot 2^x$

$2^{2x} - 4 \cdot 2^x - 12 = 0$ ✔

$(2^x + 2)(2^x - 6) = 0$ ✔

$\therefore 2^x = -2$ or $2^x = 6$

no soln ✔ $x = \dfrac{\log 6}{\log 2}$

$= 2{,}58$

✔

5

1.2. $2y - x = 3$ ... 1

$x^2 - 3xy - y^2 = 27$ ... 2

$2y - 3 = x$ ✓

$(2y-3)^2 - 3(2y-3)y - y^2 = 27$ ✓

$4y^2 - 12y + 9 - 3(2y^2 - 3y) - y^2 = 27$

$4y^2 - 12y + 9 - 6y^2 + 9y - y^2 = 27$

$0 = 3y^2 + 3y + 18$

$0 = y^2 + y + 6$ ✓ $\div 3$

$= (\quad X \quad)$?

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(6)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{-23}}{2}$

No soln ✓

∴ no points of ∩ → 4

1.3, $\dfrac{3^{3001} \cdot 3^2}{27^{1001} - 3^{3002}}$

$= \dfrac{3^{3001} \cdot 3^2}{(3^3)^{1001} - 3^{3002}}$

$= \dfrac{3^{3001} \cdot 3^2}{3^{3003} - 3^{3002}}$ ✓ pb

$= \dfrac{3^{3001} \cdot 3^2}{3^{3002}(3-1)}$ ✓ ✓ cf

$= \dfrac{3}{2}$ ✓ → 4

2.1. $30 + 28 + 26 + \cdots - 18$

$-2 \quad -2$

$T_n = a + (n-1)d$

$= 30 + (n-1)(-2)$ ✓

$= 30 + (-2n+2)$

$= 30 - 2n + 2$

$= 32 - 2n$

$\therefore -18 = 32 - 2n$ ✓

$n = 25$

So $\sum_{n=1}^{25} (32 - 2n)$ ✓ → 3

2.2. $S_n = 3^{n+1} - 6$

2.2.1. $S_{12} = 3^{12+1} - 6$

$= 1\,594\,317$ ✓ → 1

2. T_{12}

$= S_{12} - S_{11}$

$= 1594317 - (3^{11+1} - 6)$

$= 1594317 - (531435)$ ✓

$= 1\,062\,882$ → ✓ 2

2.3. $S_n = \frac{n(2a + (n-1)d}{2}$

$0 = \frac{5(2a + 4d)}{2}$

$\times \frac{2}{5}: \quad 0 = 2a + 4d$

$-2d = a$ ✓

$T_n = a + (n-1)d$

$4 = a + 4d$ ✓

$\therefore 4 = -2d + 4d$ ✓

$4 = 2d$

$2 = d$ ✓

$\therefore a = -2(2)$

$= -4$ ✓ → 5

2.4.

j j j

$-15 \quad -29 \quad -43$

$-14 \quad -14$

$d_2 = 2a \qquad d_1 = 3a + b$

$-14 = 2a \qquad -15 = 3(-7) + b$ ✓

$-7 = a \qquad 6 = b$ ✓

$T_{30} = a(30)^2 + b(30) + c$

$-6102 = 900(-7) + 30(6) + c$ ✓

$18 = c$ ✓ → 6

$\therefore T_n = -7n^2 + 6n + 18$ → −1 no conclusion

2.5. 17 15 13 ?

3 $-\frac{3}{2}$ $\frac{3}{4}$ $-\frac{3}{512}$

2.5. 1. $3 - \frac{3}{2} + \frac{3}{4} \cdots - \frac{3}{512}$

$-\frac{1}{2}$ $-\frac{1}{2}$

$T_n = ar^{n-1}$

$-\frac{3}{512} = 3\left(-\frac{1}{2}\right)^{n-1}$ ✓

$-\frac{1}{512} = \left(-\frac{1}{2}\right)^{n-1}$

$n-1 = \frac{\log\left(\frac{1}{512}\right)}{\log\left(\frac{1}{2}\right)}$ ✓

$= 9$

$\therefore n = 10$ ✓

$\therefore$ 20 terms ✓ 4

2. $17 + 15 + 13 + \cdots$

-2 -2

$S_n = \frac{n(2a+(n-1)d)}{2}$

$S_{10} = \frac{10(2(17)+9(-2))}{2}$ ✓

$= 80$ ✓

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{10} = \frac{3\left(\left(-\frac{1}{2}\right)^{10}-1\right)}{-\frac{1}{2}-1}$ ✓

$= \frac{1023}{512}$ ✓ 2

$\therefore S_{20} = 80 + \frac{1023}{512}$

$= \frac{41\,983}{512}$ ✓ 82 5

2.6. $6 ; 6 \times \frac{9}{10} ; 6 \times \frac{9}{10} \times \frac{9}{10}$

$6 ; \frac{27}{5} ; \frac{243}{50} ; \cdots$

$\frac{9}{10}$ $\frac{9}{10}$

$S_\infty = \frac{a}{1-r}$ ✓

$= \frac{6}{1-\frac{9}{10}}$ ✓

$= 60$ m ✓ 3

2.7. $1,\dot{3}\dot{6} = 1 + 0,\dot{3}\dot{6}$

Now, $0,\dot{3}\dot{6}$

$= 0,36$

$+ 0,0036$

$+ 0,000036$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{0,36}{1-\frac{1}{100}} \quad \checkmark a \quad \checkmark r$$

$$= \frac{4}{11}$$

$$\therefore 1,3\dot{6} = 1 + \frac{4}{11} = \frac{15}{11} \checkmark \rightarrow$$

3

2.8. $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$ ✓

✓ $rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$

✓ $rS_n - S_n = -a + ar^n$

$S_n(r-1) = ar^n - a$ ✓cf's

$= a(r^n - 1)$

$$\therefore S_n = \frac{a(r^n-1)}{r-1} \rightarrow$$

4

3.1. $A = P(1-i)^n$ ✓

$95\,000 = 240\,000(1-\frac{8}{100})^n$

$\frac{19}{48} = (\frac{23}{25})^n$

$$n = \frac{\log(\frac{19}{48})}{\log(\frac{23}{25})} \checkmark$$

$= 11,11$ ✓ years →

(12 full years) **3**

3.2.1. $1\,300\,000 - \frac{20}{100} \cdot 1\,300\,000$

$= 1\,300\,000 - 260\,000$

$= 1\,040\,000$ ✓

$$P = \frac{x[1-(1+i)^{-n}]}{i} \checkmark$$

$$1040\,000 = \frac{x[1-(1+\frac{12}{1200})^{-240}]}{\frac{12}{1200}} \checkmark$$

$10400 = x \cdot 0,90\ldots$

$x = R\ 11\,451,30$ ✓ →

4

2. Total

$= 260\,000 + 240 \times 11\,451,30$

$= 260\,000 + 2\,748\,312$

$= R\ 3\ 008\ 312$ ✓ →

1

3.2. 3. $P = \frac{x[1-(1+i)^{-n}]}{i}$ ✓

OB $= \frac{11451,30\left[1-\left(1+\frac{12}{1200}\right)^{-120}\right]}{\frac{12}{1200}}$ ✓

$= R\ 798\ 161,59$ ✓ 3

(OR)

$A = P(1+i)^n$

$= 1040\,000\left(1+\frac{12}{1200}\right)^{120}$

$= 3\ 432\ 402,37\ldots$ ✓

$F = \frac{x[(1+i)^n-1]}{i}$

$= \frac{11451,30\left[\left(1+\frac{12}{1200}\right)^{120}-1\right]}{\frac{12}{1200}}$

$= 2\ 634\ 242,04\ldots$ ✓

$\therefore$ OB

$= A - F$

$= R\ 798\ 160,33$ ✓ 3

3.3. 6 month $= \frac{1}{2}$ yr

$F = \frac{x[(1+i)^n-1]}{i}$

$= \frac{400\left[\left(1+\frac{14}{200}\right)^{18}-1\right]}{\frac{14}{200}}$ ✓ i+Subs ✓ n

$= 13\,599,61\ldots$ ✓

but :

$A = P(1+i)^n$

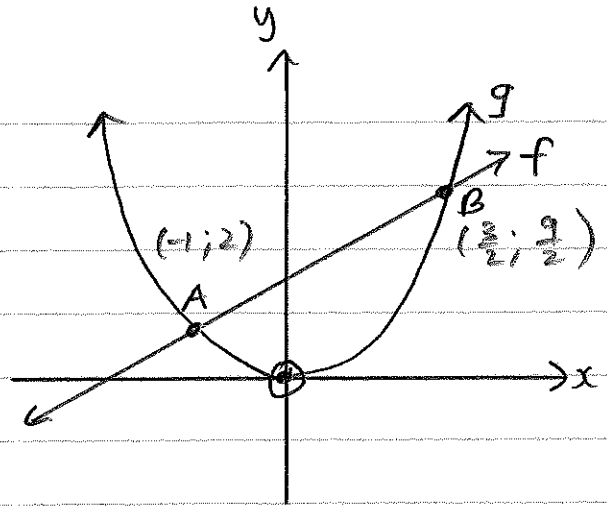
$= 7000\left(1+\frac{14}{200}\right)^{18}$ ✓

$= 23\,659,52\ldots$

$\therefore$ Value of savings

$= F + A$

$= R\ 37\ 259,14$ ✓ 5

4. 

$f: y = 2x^2$ $g: y = x+3$

4.1. Solving Sim:

$2x^2 = x+3$ ✓

$2x^2 - x - 3 = 0$ ✓

$(2x-3)(x+1) = 0$ ✓

$\therefore x = \frac{3}{2}$ or -1 ✓

$\therefore y = \frac{3}{2}+3$ or $-1+3$

$= \frac{9}{2}$ ✓ $= 2$ ✓

A(−1; 2) and B$(\frac{3}{2};\frac{9}{2})$ 6

4.2. A′(2; −1) and B′$(\frac{9}{2};\frac{3}{2})$ ✓ ✓ 2

4.3. $g: y = x+3$

$g^{-1}: x = y+3$ ✓

$x-3 = y$ ✓ 2

4.4. $f: y = 2x^2$

$f^{-1}: x = 2y^2$

$\frac{1}{2}x = y^2$

$\pm\sqrt{\frac{1}{2}x} = y$ ✓ ✓ 2

4.5. see d/s A

4.6. 1. f ↑ and g ↑

L → R

$x \in (0;\infty)$ ✓val ✓not 2

2. $f(x).g(x) \leq 0$

$y_f . y_g \leq 0$

g: x int: $0 = x+3$

$-3 = x$

$\therefore x \in (-\infty;-3]$ or $x = 0$ ✓ ✓ 2

3. $x \leq 0$ or $x \geq 0$ ✓✓ ✓✓ 2

4.7. (−1; 2) (0; 0)

av grad $= \frac{\Delta y}{\Delta x}$

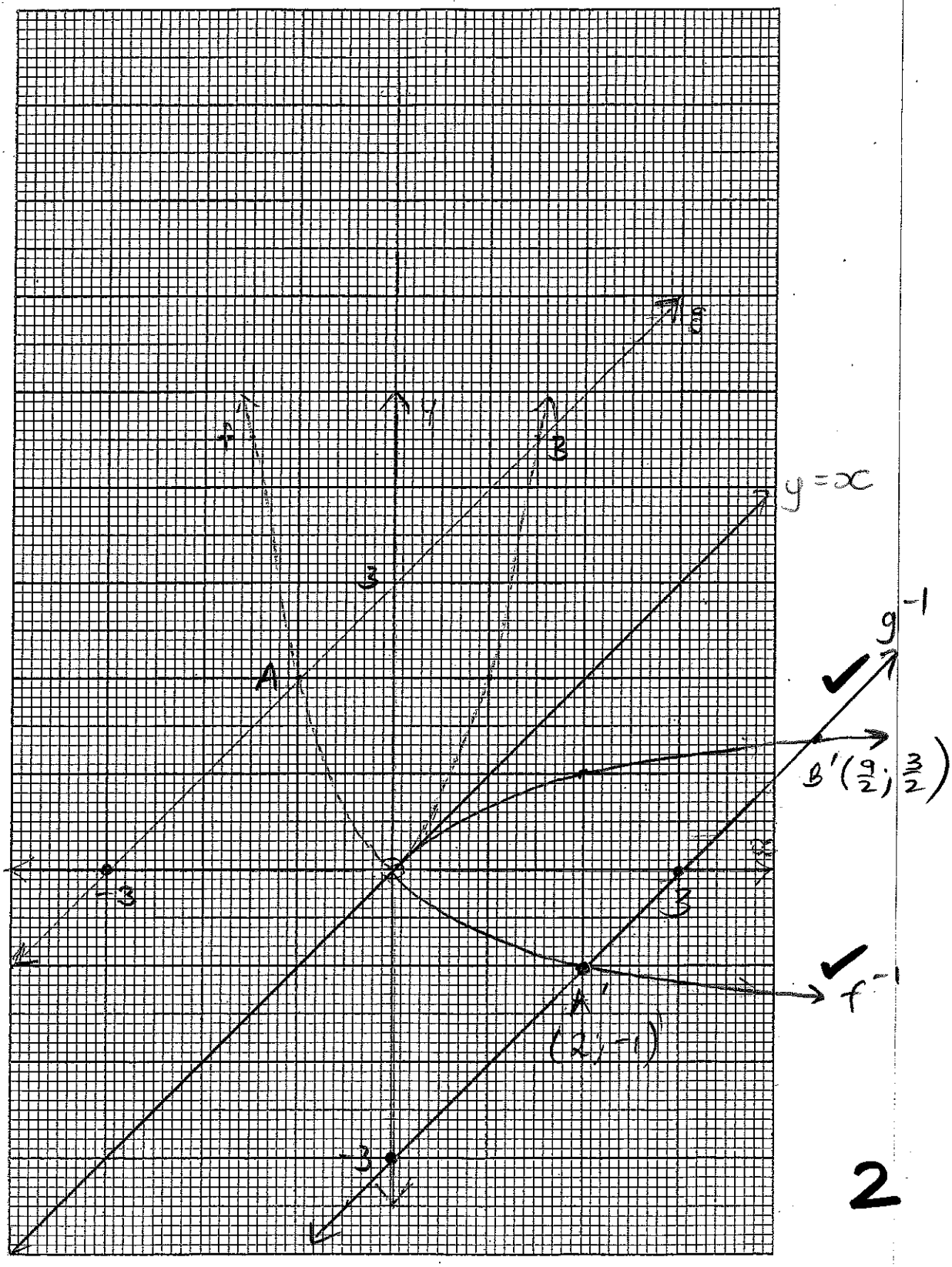
$= \frac{2-0}{-1-0}$ ✓

$= -2$ ✓

2

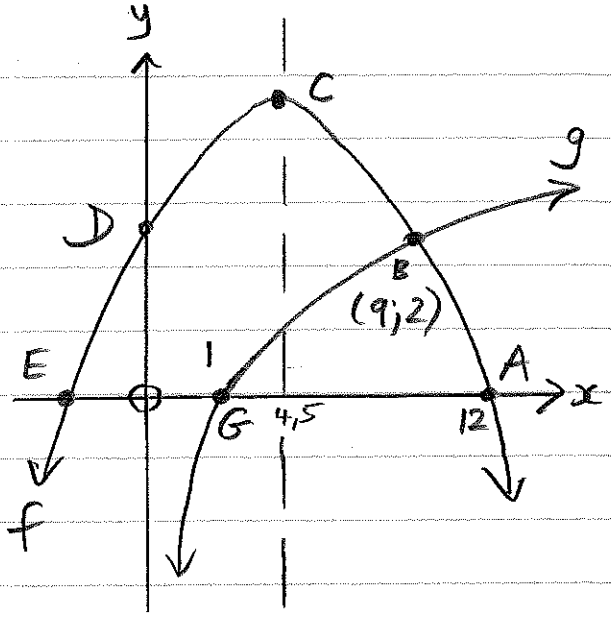
Diagram Sheet A

4.5



2

5.



f: $y = ax^2 + bx + c$

g: $y = \log_m x$

5.1. $y = \log_m x$

sub B(9;2)

$2 = \log_m 9$ ✓

$m^2 = 9$

$m = \pm 3$

$\therefore m = 3$ ✓ reject −

2

5.2. $G(1;0)$ ✓✓ 2

5.3. D_g : $x \in (0;\infty)$ ✓val ✓notn 2

5.4. g: $y = \log_m x$ ✓

g^{-1}: $x = \log_m y$ ✓

$m^x = y$ → 3^x

2

5.5. g^{-1} $y = m^x$ ✓

2 ← $y = m^{x+2}$ → 1

5.6.1. AoS: $x = 4,5$

$\therefore \frac{12 + x_E}{2} = 4,5$

$\times 2$: $12 + x_E = 9$ ✓

$x_E = -3$

$\therefore E(-3;0)$ 1

2. $y = a(x+3)(x-12)$ ✓

sub E(9;2)

$2 = a(9+3)(9-12)$ ✓

$2 = a(12)(-3)$

$= a(-36)$

$\frac{2}{-36} = a$

$-\frac{1}{18} = a$

$\therefore y = -\frac{1}{18}(x+3)(x-12)$

$= -\frac{1}{18}(x^2 - 9x - 36)$ ✓

$= -\frac{1}{18}x^2 + \frac{1}{2}x + 2$

$\therefore a = -\frac{1}{18}$, $b = \frac{1}{2}$ and $c = 2$

3

5.7. $y = -\frac{1}{18}x^2 + \frac{1}{2}x + 2$

Don't say by cts

$\therefore p = -\frac{b}{2a}$

$= \frac{-(\frac{1}{2})}{2(-\frac{1}{18})}$

$= \frac{9}{2}$ ✓

$q = -\frac{1}{18}(\frac{9}{2})^2 + \frac{1}{2}(\frac{9}{2}) + 2$

$= \frac{25}{8}$ ✓

✓ $y = -\frac{1}{18}(x - \frac{9}{2})^2 + \frac{25}{8}$

3

5.8. $f(x) - 4 = 0$

$y_f - 4 = 0$

$y_f = 4$

max $y_f = \frac{25}{8}$ $(a<0)$

< 4

So $y_f \neq 4$

ie $y_f - 4 = 0$

has no real roots / soln

✓✓ 2

or any logical alt.

6.1. $f: y = \frac{-3}{x+2} + 3$

y int: $y = \frac{-3}{0+2} + 3$

$= \frac{3}{2}$ 1,5

x int: $0 = -\frac{3}{x+2} + 3$

$\frac{3}{x+2} = 3$

LCD $x+2$ $(\therefore x \neq -2)$

$x \neq m$

$3 = 3(x+2)$

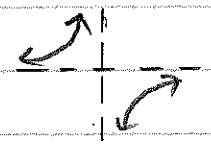
$-1 = x$

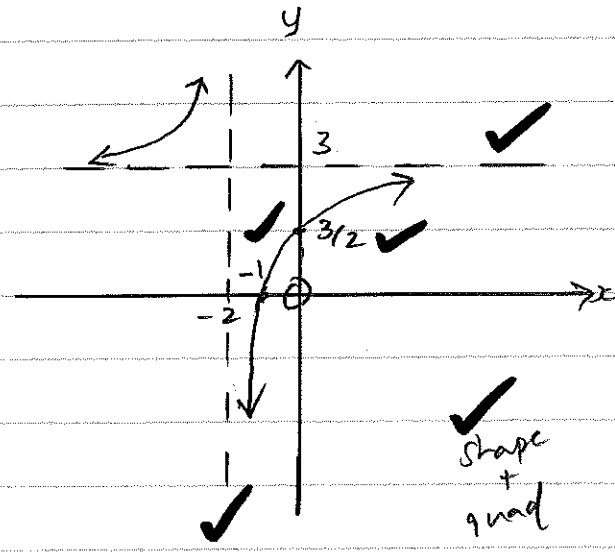
ha: $y = 3$

va: $x + 2 = 0$

$\therefore x = -2$

shape $k = -3$





✓ ✓ ✓ ✓ ✓ shape + quad

5

6.2. 1. $y = x+2+3$

$= x+5$ ✓

and

$y = -(x+2)+3$

$= -x-2+3$

$= -x+1$ ✓

2

2. ✓ ✓ $x = -2$ and $y = 3$

2

6.3. D_f: ✓✓ $x \in \mathbb{R};\ x \neq -2$

2

7.1. $h(x) = x^3 + (q-4)x^2 + (3-4q)x + 3$

$h(1)^{x=1} = (1)^3 + (q-4)(1)^2 + (3-4q)(1) + 3$

0 ✓ $= 1 + q - 4 + 3 - 4q + 3$

$3q = 3$

$q = 1$ ✓

3

7.2. 1. $f(x) = 2x^3 + x^2 - 5x + 2$

$2x - 1 = 0 \quad \therefore x = \frac{1}{2}$

✓

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(\frac{1}{2}) + 2$

$= 0$ ✓

$\therefore$ $2x-1$ is a factor

2

2. $2x^3 + x^2 - 5x + 2$ ✓

$= (2x-1)(x^2 + x - 2)$ ✓

$-x^2$

$+2x^2$

$= x^2$ ✓

$= (2x-1)(x+2)(x-1)$

3

8.1. $f(x) = \frac{3}{x}$

$f(x+h) = \frac{3}{x+h}$

$f'(x)$

$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$ ✓

$= \lim_{h \to 0} \left(\frac{3x - 3(x+h)}{(x+h)x} \right) \div h$ ✓

$= \lim_{h \to 0} \left(\frac{3x - 3x - 3h}{(x+h)x} \right) \times \frac{1}{h}$

$= \lim_{h \to 0} \frac{-3h'}{x(x+h)} \times \frac{1}{h'}$

$= \lim_{h \to 0} \frac{-3}{x(x+h)}$ ✓

$= \frac{-3}{x(x+0)}$

$= -\frac{3}{x^2}$ ✓ 6

8.2. $f'(3) = -\frac{3}{(3)^2}$ ✓

$= -\frac{1}{3}$ ✓

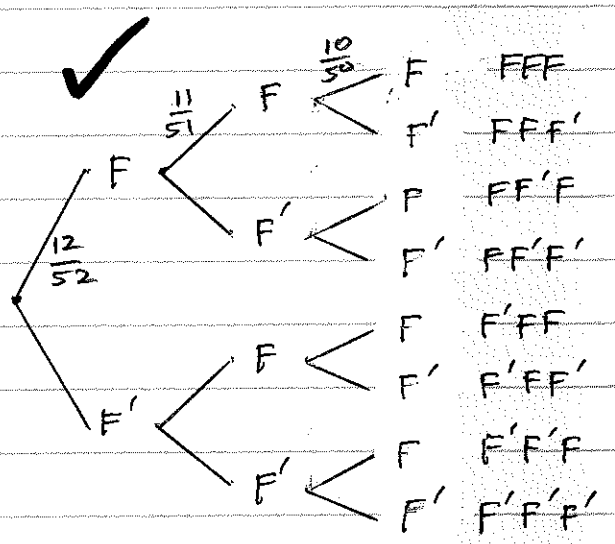
The instantaneous gradient of f when $x = 3$ is $-\frac{1}{3}$ ✓

= grad of tan line to f when $x = 3$

3

(11)

9. ✓



P(FFF)

$= \frac{12}{52} \times \frac{11}{51} \times \frac{10}{50}$ ✓

$= \frac{11}{1105}$ ✓

0,01

5

F = Face card

F' = not Face card